**INTRODUCTION**

Goods that are shipped world-wide travel protected by packages that are frequently made of corrugated board. Therefore, the strength of corrugated board containers is crucial for preserving the content, while an optimization of corrugated board containers is essential to save money and resources.

Unfortunately, standard compression tests show a wide dispersion in container performance, and so, to avoid failure often containers have to be overdesigned. Dispersion in performance arises from different factors, among which the most important are variable quality of raw materials, imperfections produced by the manufacturing process in local and global geometry, and environmental conditions. Standard quality tests performed on the finished container are affected simultaneously by influencing parameters making identification of the cause of the lack of strength a difficult task.

Structural analysis of paperboard components is a critical topic in the design of containers that have to withstand compression loads due to stacking. The evaluation of a container’s buckling behaviour requires a deep knowledge of paperboard stiffness properties.

In the reviewed literature we found studies on the strength of paperboard, including the effect of local buckling exhibited by the liners. However, in this study elastic properties are evaluated using composite laminate plate theory. A numerical procedure was proposed to evaluate the global and local instability of paperboard sheets loaded in both the machine and the cross-direction, taking into account the actual microstructure, with a simplified tooth-shaped geometric model.
A complete design procedure for a paperboard shelter was described, assuming classical lamination theory to be applicable. For the structural general behaviour, experimental and numerical evaluation of transverse shear stiffness and bending stiffness by means of standard TPB and shear tests were presented, in which numerical analyses were successfully conducted on local geometry by means of the finite element method, representing the corrugated medium with a sine wave, but limiting numerical part as a tool to better understand the experimental results; a numerical evaluation addressed to the complete determination of flexural properties of corrugated board panels was reported, modelling the corrugated medium with a trapezoidal wave; recently an experimental evaluation of flexural stiffness matrix by mode recognition of an orthotropic plate was presented. An experimental and numerical investigation of transversal crushing was reported, considering also the effects of manufacturing-induced geometrical imperfections. A non-linear analysis of the compression of corrugated board containers was presented, performing a comparison between predicted and experimental data.

Besides the scientific contributions strictly addressed to corrugated board, several studies on corrugated panels can be found for corrugated decks: first, a contribution about a corrugated sheet; second, a closed form to evaluate equivalent properties for corrugated panels is proposed, and similar closed-form calculations can be found; the authors have extended the closed form solution, applicable for corrugated sheets made by isotropic material, to the analysis of corrugated board.

The present study proposes an experimental and numerical approach suitable for systematically determining corrugated board behaviour, in which empirical assumptions are replaced by the mechanical and micro-mechanical behaviour of corrugated board; an advanced tool design is proposed that could be exploited to optimize the final product, controlling all the input parameters. In order to achieve this goal, standard production tests, such as the edge compression test and the box compression test, were completed by tests on the raw materials, starting from the experience and empiricism of the corrugated board manufacturing world, to reach a deep insight in corrugated board mechanics.

As depicted in Figure 1, corrugated board is a composite structure in which a corrugated core, the fluting, is bonded between two paperboard sheets, the liners, by means of an adhesive film. In order to predict the overall structural behaviour of corrugated board, a mechanical characterization of the raw materials, i.e. paper sheets, is needed. The first step is, then, the evaluation of the elastic constants and strength of anisotropic paper sheet that could be effectively modelled as an orthotropic lamina in plane stress with four parameters, $E_L$, $E_T$, $v_{LT}$ and $G_{LT}$, where the subscript L refers to the longitudinal direction, or machine direction, in which the fibres are preferentially oriented during paper manufacturing, and the subscript T refers to the transverse direction, or cross-direction.

A full detailed finite element model was developed to reproduce a region of local geometry.
After mesh size optimization, numerical model correctness was checked by simulating a standard edge compression test (ECT) and comparing the experimental data with the numerical results.

A rectangular region of corrugated board was then modelled to perform a series of virtual characterization tests to extract the in-plane and out-of-plane elastic constants. Starting from these constants obtained on the full detailed model, an equivalent shell element ‘corrugated board’ was defined.\textsuperscript{22,23} Numerical parameter evaluation was conducted, together with a theoretical evaluation based on classical lamination theory.

This approach is better than a super-element approach based on a simple static condensation, because with a super-element, a numerical matrix dependent on condensed region dimension results, while the new homogenized element will still be based on general shell theory, producing a parametric stiffness matrix, together with the geometric matrix for non-linear analyses; furthermore, membrane forces and bending moments acting on the condensed element can be transferred to the microstructural model to perform local stress analysis.

Finally a complete corrugated board container was modelled with homogenized elements, reducing by three orders of magnitude the degrees of freedom necessary for the analysis. A box compression test (BCT) was simulated and the numerical results were compared with experiments. It is important to notice that comparison are valid only for the incipient buckling load predicted with the numerical analysis and observed during the BCT test. In fact, the maximum compression load, which was not investigated numerically in this paper, depends on the postbuckling behaviour of the structure and is, for the sample experimentally tested, about twice the incipient buckling load (see Table 5).

\section*{EXPERIMENTAL}

First, several paperboards for liners and fluting were tested to obtain directional strength and stiffness, then ECT and BCT tests were conducted for various corrugated boards. All tests were conducted using an electromechanical Universal Testing Machine, Intron Series IX, controlled by a personal computer using Merlin software to manage the test parameters and extract the desired outputs. According to the standards, all measurements were conducted in a conditioned environment at $23 \pm 2^\circ C$ with a relative moisture level of $50 \pm 2\%$.

\textbf{Paperboard characterization}

Standard paper tests were conducted to evaluate surface density, thickness, Young’s and Poisson’s moduli, yielding and ultimate stresses, for various materials suitable for the production of compression containers.\textsuperscript{19,20} As explained in the introduction, due to a preferential fibre orientation and small thickness, paper sheet could be modelled as an orthotropic material in plane stress. Symmetry directions were determined by the manufacturing process that orientates the fibres in the machine direction. Thickness and density were evaluated using a round quadrant thickness tester and a precision balance. The stiffness matrix evaluation requires four independent parameters, $E_L$, $E_T$, $v_{LT}$ and $G_{LT}$:

$$\begin{bmatrix}
K
\end{bmatrix} = \begin{bmatrix}
E_L^3 & v_{LT}E_LE_T & 0 \\
E_Lv_{LT}E_T & E_L - v_{LT}^2E_T & v_{LT}E_LE_T \\
v_{LT}E_LE_T & v_{LT}E_LE_T & E_L - v_{LT}^2E_T \\
0 & 0 & G_{LT}
\end{bmatrix} \quad (1)
$$

Young’s and Poisson’s moduli, $E_L$, $E_T$, $v_{LT}$, were measured by means of simple tensile testing on specimens orientated in the machine direction and the cross-direction; the shear modulus was evaluated by means of a third series of tensile tests for specimens orientated at $45^\circ$ transverse direction, in order to evaluate $G_{LT}$ from $E_\theta$ according to the following equation, obtained for the $45^\circ$ rotated stiffness matrix:

$$G_{LT} = \left[ \frac{2v_{LT}}{E_L} - \frac{1}{E_L} - \frac{1}{E_T} + 4 \right]^{-1} \quad (2)
$$

According to the Italian standard UNI EN ISO 1924–2,\textsuperscript{24} rectangular specimens of $180 \times 15 \text{ mm}$ were tested under constant displacement velocity of $20 \pm 5 \text{ mm/min}$, repeating the measure for at
least 10 specimens for each material. Figure 2 shows load vs. displacement curves recorded for a KL6 (designated according to the Italian standard GIFCO) paperboard, together with linear fit results adopted for Young’s modulus evaluation.

Poisson’s moduli were evaluated with image processing, acquiring a magnified image of the specimen before and after an impressed displacement of 0.7 mm in the machine direction i.e. small enough to still have linear behaviour but compatible with optical processing resolution. The measurements were repeated for five specimens, evaluating longitudinal and transversal strain of a square target (10 mm sided) printed in the central region of each specimen, and computing the Poisson’s modulus using the formula:

\[ v = -\frac{\varepsilon_l}{\varepsilon_t} \]  

(3)

Paper testing was completed with ring crush tests (RCT). According to the standard TAPPI T.818,25 a paper strip 152.4 × 12.7 mm was inserted into a cylindrical metallic support to avoid local buckling, and compressed between two rigid platens at a constant speed of 12.5 ± 2.5 mm/min in a standard environment, until critical load was reached. The same measurements were repeated for at least 10 specimens. As explained later, in the Discussion, the linear combination of RCT of fluting and liners controls corrugated board ECT and BCT by the McKee relation (see equation 9).

Corrugated board

Our experiments were addressed to a single-walled symmetrical board, KL-S-KL 595 C (according to the GIFCO designation, this corrugated board consists of two KL-5 liners and a C type S-9 fluting), being a standard for the production of high strength compression containers. The edge compression test (ECT) was performed according to standard FEFCO No. 8, on rectangular specimens 100 mm × 25 mm inserted between two compression platens, loaded in a standard environment at constant speed until instability occurred.

The ECT value is an important indicator of corrugated board quality and is widely used for quality control and for container design via the McKee formula, which estimates BCT values depending on the container geometry and ECT.

Containers

Containers were subjected to the box compression test (BCT). According to FEFCO standard No. 50, at least five specimens were compressed between two platens until maximum load was reached, at a constant speed of 10 mm/min. BCTs were performed with the same universal testing machine adopted for paper testing, recording load vs. displacement curves.

In order to handle a statistically relevant population, the test was repeated on 10 specimens with
dimensions 385 × 293 × 350 mm, made with corrugated board KL-S-KL 595C. Furthermore, to evaluate the effect of the container manufacturing process, the experiments were conducted on production containers and on standard specimens assembled in the laboratory using a dedicated CAD cutter and using a virgin corrugated board panel.

Obviously, the standard specimens have the best performances achievable with a specific corrugated board, because the automatic manufacturing process damages the container.

**NUMERICAL MODELS**

To model the structural behaviour of corrugated board packages, several finite elements models were developed. First, the local geometry was analysed with a full detailed model, in which paperboard sheets for liners and fluting were modelled with isoparametric four-node shell elements with a two-dimensional orthotropic material rule; then an equivalent ‘corrugated board’ element, able to reproduce, with a single element, the behaviour of the modelled region and suitable for the FEM analysis of global geometry at the container detail level, was developed.

**Local model**

FEM analyses were performed with the commercial code MSC/Nastran, version 70.5, and the preprocessor FEMAP 7.0. Local geometry of corrugated board KL-S-KL 595C was acquired from the photographic magnification of the cross-section of an actual specimen with thickness 4.1 mm, corrugation length 8 mm and corrugation factor 1.435. Figure 3 shows a diagram comparing the actual shape and a cosine shape.

Shell elements of the model were properly orientated to reproduce the actual pattern of paperboards of liners and fluting. A rectangular corrugated board region was modelled according to the standard dimension of the ECT specimen (100 mm × 25 mm); the FEM model is represented in Figure 4.

For ECT simulation, a load was introduced on a master node on the upper side, slaving the vertical displacements of the nodes on the upper face, while the lower face nodes were constrained, suppressing the vertical displacements. The upper face was loaded with 500 N, and the first collapse eigenvalue was extracted using stan-
standard NASTRAN solution sequence SOL 105 (SEBUCKL). Mesh size was adjusted to reach a convergent eigenvalue; the convergent FEM model consisted of 4225 shell elements CQUAD4 and 4251 nodes; and the convergent results allowed estimation of a numerical ECT of 7.85 kN/m. In order to evaluate the applicability of eigenvalue solution, stress results at critical value were computed, verifying that the maximum Von Mises stress recorded (6.5 MPa in the cross-direction for KL5 material) was smaller than the yielding stress (7.07 MPa in the cross-direction, as shown in Table 2).

### Equivalent stiffness for ‘corrugated board’ element

In order to evaluate equivalent stiffness of corrugated board KL-S-KL 595C, a rectangular model 32 mm × 25 mm was extracted from the FEM model that was already optimized for ECT simulation. The modelled region, consisting of 1167 nodes and 1126 elements CQUAD4, was subjected to in-plane shear and tensile stress and off-plane flexure and torsion, with the aim of a direct evaluation of matrix $ABD$ (see equation 4) for the corrugated board region. Where applicable, classical lamination theory was adopted to verify numerical model results.

Stiffness homogenization was performed for an equivalent shell region with the same in-plane dimension and with a reduction in stiffness of 4 mm.

According to equation 4 and to the notation of Figure 1, $\varepsilon_i$ are in-plane extensional ($i = 1,2$) and shear ($i = 6$) strains, $k$ are shell curvatures $[A], [D]$ and $[B]$ are $3 \times 3$ matrices, representing extensional stiffness, flexural stiffness, and coupling between in-plane and off-plane behaviour.

$$
\begin{pmatrix}
N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6
\end{pmatrix} =
\begin{bmatrix}
[A] & [B] \\
[B] & [D]
\end{bmatrix}
\begin{pmatrix}
\varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \\ \kappa_1 \\ \kappa_2 \\ \kappa_6
\end{pmatrix}
$$

(4)

Stiffness parameters were extracted with a compliance method assigning six load and boundary conditions, one for each load vector component, and processing the results to obtain engineering constants.

For the investigated board, the $[B]$ matrix is zero for symmetry, as confirmed by the static analyses performed.

<table>
<thead>
<tr>
<th>Name</th>
<th>$\sigma_T$ (MPa)</th>
<th>Error (%)</th>
<th>$\sigma_{TS}$ (MPa)</th>
<th>Error (%)</th>
<th>$\sigma_L$ (MPa)</th>
<th>Error (%)</th>
<th>$\sigma_{IL}$ (MPa)</th>
<th>Error (%)</th>
</tr>
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<tbody>
<tr>
<td>S-9</td>
<td>6.36</td>
<td>22.2</td>
<td>14.91</td>
<td>6.17</td>
<td>10.9</td>
<td>16.3</td>
<td>32.6</td>
<td>8.5</td>
</tr>
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<td>KL-5</td>
<td>7.07</td>
<td>19.4</td>
<td>19.4</td>
<td>5.3</td>
<td>10.4</td>
<td>13.4</td>
<td>41.2</td>
<td>4.0</td>
</tr>
<tr>
<td>KL-6</td>
<td>8.3</td>
<td>17.0</td>
<td>20.7</td>
<td>2.84</td>
<td>10.3</td>
<td>14.9</td>
<td>41.4</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Table 2. Experimental results for corrugated board paper designated according to Italian standard GIFCO1: Strength

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In-plane stiffness. In-plane stiffness parameters were extracted using two tensile tests and a shear test, looking for the elastic constants of a homogeneous orthotropic material characterizing the in-plane behaviour of the equivalent element at the prescribed reduction thickness (see Figure 1 for notation). As shown in Figure 5, tension in the direction ‘N2’ (cross-direction) was imposed, applying a vertical load on the upper face, restraining vertical displacements on the face to the same value by means of a rigid element. The resulting displacement allowed extraction of a Young’s modulus for a three-dimensional model of 410.3 MPa, computed at the reduction thickness of the equivalent element. Poisson’s modulus was computed from lateral contraction to be 0.218.

The same result can easily be computed by classical lamination theory according to matrix A definition:

\[
A_{ij} = \sum_{k=1}^{n} (z_k - z_{k-1}) Q_{ij}^{(k)} = \sum_{k=1}^{n} s_k Q_{ij}^{(k)}
\]

where \( k \) designates one of the \( n \) plies, \( h \) and \( s_k \), respectively, overall thickness and ply thickness, and \( Q_{ij} \) are the coefficients of the material stiffness matrix. With the elastic moduli in machine direction at 1694.0 MPa for the KL-5 liners and 1532.0 MPa for S-9 fluting, and considering the actual cross-section of fluting, a theoretical value of 410.15 MPa, which strictly agrees with the numerical results, is predicted.

Following the same procedure for the machine direction a Young’s modulus of 495.3 MPa was obtained by the numerical model, and a lateral contraction factor of 0.301. Theoretical estimation for MD was conducted, neglecting the contribution of the fluting obtaining a value of 482.0 MPa.

Shear test was performed by constraining the displacements of upper and lower faces and imposing 1 mm of horizontal displacement for the nodes of the upper face, as shown in Figure 5. The resulting horizontal load was obtained by adding all the reactions; shear modulus was computed to be 117.2 MPa.

The overall A matrix for KL-S-KL 595C corrugated board is as follows:

\[
A = \begin{pmatrix}
2.131 & 0.539 & 0 \\
0.539 & 1.798 & 0 \\
0 & 0 & 0.468
\end{pmatrix} \text{MPa} \cdot \text{m}
\]

Flexural stiffness. As for in-plane stiffness, flexural parameters were extracted in order to obtain a homogeneous orthotropic material characterizing the flexural behaviour of the equivalent element at the prescribed reduction thickness (see Figure 1 for notation).

Neglecting shear terms and using orthotropic plate theory, the vertical displacements could be expressed by equation 6:
which expresses the displacement fields in terms of Young’s and Poisson’s moduli in the X direction (but is still valid for the Y direction by changing the subscripts) and overall thickness, and is suitable for extracting stiffness parameters from the three-dimensional model matching displacement field resulting from FEM analysis.

Flexural stiffness terms could also be evaluated by classical lamination theory, as follows:

\[ D_{ij} = \frac{1}{2} \sum_{k=1}^{n} (z^2_i - z^2_{i-1}) Q_{ij}^{(K)} \]  

where \( D_{ij} \) are flexural stiffness terms, \( z_i \) and \( z_{i-1} \) are thickness direction positions of each lamina \( k \), \( h \) is the overall thickness and \( Q_{ij} \) are the material stiffness terms for lamina \( k \).

As illustrated in Figure 6, the nodes at the left side of the rectangular region were constrained only in the horizontal direction, and the moment was applied by means of a rigid element connecting the horizontal displacements of the nodes at the right side. A deformed shape with two curvatures results: the first in the load direction, the second in transverse direction, due to Poisson’s effect. Matching the numerical displacements with the closed-form solution for the orthotropic plate, Young’s and Poisson’s moduli can be extracted.

For the machine direction, equivalent moduli are 1129.1 MPa and 0.27 and a rough estimation of fluting stiffness was carried out, considering that a generic transverse section of the fluting has a rectangular shape, and the distance from middle plane varies with the longitudinal coordinates; an equivalent moment of inertia could be computed, averaging the variable moment of inertia in a complete period, obtaining for the equivalent modulus a value of 1315 MPa, which is a 16% overestimation of the numerical predicted value.

Following a procedure similar to that adopted for in-plane shear, torsion stiffness was extracted by loading the three-dimensional model as a beam (see Figure 6), constraining a face and loading the opposite one with a unit moment, introduced with a rigid element. Matching the displacement, we get a shear modulus for flexural behaviour of 191.5 MPa. The overall \( D \) matrix for KL-S-KL 595C corrugated board is as follows:

\[
D = \begin{pmatrix}
6.327 & 1.131 & 0 \\
1.131 & 4.188 & 0 \\
0 & 0 & 1.725
\end{pmatrix} \quad \text{Pa} \cdot \text{m}^2
\]

**Equivalent element validation.** The correctness of the detailed model reduction in a single element has to be verified before using equivalent element in further calculation. The first check has already been discussed, and is a comparison with a theoretical value when available.

The second check regards the symmetry of the matrix obtained for orthotropic material assumption; the moduli obtained have to match the following equation:

\[
\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2} \Rightarrow v_{12}E_2 = v_{21}E_1
\]  (8)
Both in-plane and flexural moduli agree, with symmetry within a 10% maximum deviation.

The last check consists of the FEM simulation of a rectangular panel, built duplicating two by two the original region, modelled first in full detail (about 4500 elements) and then by means of four condensed elements (see Figure 7).

A static load, obtained by a constant body load field equal to a $-5 \text{m/s}^2$ in the $X$ direction, $-9.81 \text{m/s}^2$ in the $Y$ direction, $5 \text{m/s}^2$ in the $Z$ direction, was used. Computed displacements and rotations in all the nodal points of the condensed model have a maximum relative difference below 4%, compared with the corresponding node in the full detailed model.

The same model was also used for a buckling analysis, loading both edges with a uniform compressive load of 10N. Eigenvalue analyses produced a critical load of 250.28N for the full model and a critical load of 276.39N for the condensed model. The relative error of 10% is probably connected with mesh coarseness of the condensed model; however, as observed in the introduction, the geometric matrix is still sufficiently accurate for non-linear geometric analysis, and also for the equivalent element.

**Analysis of a box container**

**Investigation of closure fin stiffness contribution.** A container box is quite a complex structure and may be thought of as a channel completed at both ends with four closure fins. In order to investigate the effects of the various assembling parts, a simple numerical experiment was conducted. For the sake of simplicity, an isotropic material with Young’s modulus of 2500MPa and Poisson’s modulus of 0.32 was considered for this experiment. The first geometry investigated was a single square panel, $100 \text{mm} \times 100 \text{mm} \times 1 \text{mm}$, compressed at the opposite edges. The FEM model consists of 400 four corner plate elements and was constrained only in vertical displacements at the bottom edge (with the exception of a node that was constrained also in both transversal direction and in the rotation about vertical axis, to avoid lability) and loaded on the upper edge with 10N. Buckling analysis for this panel gives the first eigenvalue at 0.18 (Figure 8).

The same analysis was performed but adding two lateral walls shaped as the starting panel and maintaining the same loading and constraint condition, obtaining the first eigenvalue at 0.205. The last analysis was performed by adding two $25 \text{mm} \times 100 \text{mm} \times 1 \text{mm}$ fins, at the up and bottom free edge of the initial panel, maintaining the same loading and constraint condition of the first analysis, obtaining the first eigenvalue at 0.476.

This simple study suggests that, to model the overall instability of a box, closure fins have to be accounted for, because they introduce a constraining effect that imposes a parabolic buckling eigenvector of the vertical walls, similar to a plate elastically clamped at the four sides.

**BCT numerical simulation.** A complete container $385 \times 293 \times 350 \text{mm}$ was modelled by means of 927 KL-S-KL 595C corrugated board elements and 999 nodes; the same geometry modelled in full detail would required about $1.04 \times 10^6$ elements and $1.08 \times 10^6$ nodes. The characteristic dimension of the elements is similar, but not equal, to that of the region used for equivalent element determination; obviously a finer mesh may be developed, but if equivalent elements have a characteristic
dimension comparable with the micro-geometry, the condensed model will not be applicable and local effects become dominant.

According to the BCT standard, closure fins without the adhesive tape are free to move and can interact only for mutual contact; this interaction was not considered and the FEM model, representing the open box, deforms exactly like that representing the closed box, but the results are easily interpreted (Figure 9). To correctly reproduce the action of the compression platen, the nodes on the bottom edges were constrained in vertical displacement, and the node on the vertical displacements of the node on upper edges were connected to a master node, loaded with the compression load of 100N.

A critical load of 2043.3N was then obtained by performing an eigenvalue buckling analysis; as observed for the ECT simulation, a stress analysis at this load level has to be conducted to verify that the failure mechanism can be estimated by eigenvalue buckling analysis. Stress is not directly available in the static analysis of the condensed element, and stress analysis was conducted by extracting the most loaded element in terms of flexural and tensile loads, and introducing this load into the full detailed model of the single element, obtaining a maximum Von Mises stress of $\sigma_{VM} = 1.88$ MPa that is lower than the minimum yielding stress of the box materials ($\sigma_y = 6.36$ for S9 board, as reported in Table 2).
RESULTS AND DISCUSSION

Experimental results regarding paper behaviour are reported in Tables 1 and 2, for a variety of papers used for liners and fluting of corrugated board. Each modulus value was obtained through a statistical analysis conducted on a population of 10 specimens, computing the expected value as the mean value and the error (%) from the standard deviation, and is reported in the first six columns of Table 1. As discussed above, moduli $E_L$, $E_T$ and $E_{45}$ were obtained by a simple tension test, the shear modulus was derived using equation (2), and Poisson’s ratio $v$ was estimated using an optical method. In Table 1, paper density $\rho$ and thickness are also reported; obviously, modulus estimation of each specimen was obtained considering the starting slope of force displacement curve, considering the actual specimen dimension.

Moduli in the machine direction were found to be higher than in the cross-direction for the paper anisotropy introduced in the manufacturing process; furthermore, MD values are less dispersed than CD values and this is probably due to the fibre orientation, because along the fibre direction the mechanical behaviour is less influenced than in the cross-direction, by fibre interaction and its random nature. For the shear modulus, a wider dispersion was observed ($15 \pm 18\%$) but a direct comparison with longitudinal moduli is not applicable here, because this quantity was derived by a longitudinal modulus at $45^\circ$ and so the shear modulus is masked by experimental error propagation.

Table 2 summarizes the strength parameters computed from the complete load displacements curves, reporting first the yielding and ultimate stress in the cross-direction ($\sigma_{yt}$, $\sigma_{ut}$) with their standard errors, and then the yielding and ultimate stress in the machine direction ($\sigma_{yt}$, $\sigma_{ul}$).

The ring crush test (RCT) was performed to verify the compliance of the adopted paperboard with the standard values expected. Weak RCT is a signal of bad quality due to defects or moisture; comparison between measured values and standard values is summarized in Table 3.

According to the standard, the edge compression test (ECT) was repeated 10 times for each corrugated board tested. As summarized in Table 4, an ECT value of 7.74 kN/m was obtained for specimens of corrugated board KL-S-KL 595 C extracted from virgin panels, and an ECT value of 7.48 kN/m was obtained for specimens extracted from production panels; the latter value agrees with the theoretical value of 7.475 kN/m predicted by the following design relation:

$$ECT_{theoretic} = RCT_{lin1} + RCT_{lin2} + RCT_{flu} \times C.O \quad (9)$$

where $lin1$ and $lin2$ denote inner and outer liners, $flu$ the fluting and C.O. is the wave factor (1.435 for analysed board).

The measured value of ECT (7.74 kN/m) for a virgin panel is consistent with the numerical value predicted by FEM analysis (7.85 kN/m, as reported in Local model above) with a negligible relative error (1.4%). This result confirms that the failure mechanism is handled very well by a simple eigenvalue buckling analysis and that the FEM model is representative of the virgin corrugated board, because damages introduced during the manufacturing process are neglected in the

<table>
<thead>
<tr>
<th>Paper</th>
<th>KL-3</th>
<th>KL-5</th>
<th>KL-6</th>
<th>T-5</th>
<th>S-6</th>
<th>S-9</th>
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<td>Measured value (kN/m)</td>
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<td>2.51</td>
<td>1.71</td>
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<table>
<thead>
<tr>
<th>Board</th>
<th>Status</th>
<th>ECT (kN/m)</th>
<th>Err (%)</th>
</tr>
</thead>
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<tr>
<td>KL-S-KL 595 C</td>
<td>Virgin</td>
<td>7.73</td>
<td>5.17</td>
</tr>
<tr>
<td>KL-S-KL 595 C</td>
<td>Production</td>
<td>7.48</td>
<td>6.5</td>
</tr>
</tbody>
</table>
model. Furthermore, eigenvector shape reproduces very well the observed deformed shape of specimen at critical load. If the microgeometry to be condensed in the equivalent element is modelled in a similar fashion, a result close to the actual should be expected also for a corrugated board structure.

To assess the reliability of modelling by means of corrugated board elements already described, a comparison with experimental BCT results was performed. Load vs. displacement of compression platens recorded during a BCT test are represented in Figure 10 for a best quality box and a production box.

Loading curves are divided into three stages: the first, with the lower slope, denotes the progression of contact between the platens and box edges due to the irregular geometry of the box; the second stage is nearly linear and corresponds to the compression at the value at which the geometric effects are negligible; the third region is highly non-linear and corresponds to the post-buckling stage. In the transition between the second and third regions, a local damage at one of the box angles is observed; residual stiffness is due to the contribution of non-collapsed angles and to the permanent deformation introduced in the structure; the critical and maximum loads for containers the tested are summarized in Table 5.

The FEFCO standard prescribes a preload of 20 kg (196 N) that is high enough to have a complete contact, but setting the BCT value the maximum load, neglecting the instability point calculation.

A design formula, known as McKee formula, is available also for the estimation of BCT value:

$$BCT_{theoretic} = 1.82 \times ECT \times \sqrt{S} \times \sqrt{P}$$

(10)

where $S$ is the corrugated board thickness (in mm) and $P$ is the perimeter of the box (in cm). The same
trend exhibited by ECT value is found for BCT, the value predicted by McKee formula (3.26 kN) being in good agreement with the measured value for a production box (3.44 kN).

As for ECT, the numerical values predicted by means of FEM model are expected to be representative of a standard box; however, comparison with numerical results is applicable only at the instability point because numerical prediction of post-buckling behaviour and maximum load was not taken into account with the proposed model. The numerical result for BCT (2043.3 N) was compared with the instability load recorded for the best quality box, which was 2.36 kN and differed from the predicted value by only 7%; furthermore, subtracting the preload of 196 N imposed to have complete contact, the experimental value becomes 2200 N, which is about coincident with the numerical result.

**CONCLUSIONS**

A numerical procedure was proposed for the evaluation of corrugated board panel structural performances, starting from the paper characteristics and micro-geometry. The reliability of the proposed model was checked by comparing the numerical results with the experimental ones, obtaining an excellent agreement first for the ECT test and then for the BCT test.

The results obtained show that corrugated board incipient buckling load is adequately described by a simple eigenvalue buckling analysis, based on the linear material behaviour and geometric stiffness matrix. These numerical methods were demonstrated to be suitable also for the proposed micro-geometry reduction.

This numerical tool is applicable to a wide range of problems concerning corrugated board structure, for both first design and product optimization, because all parameter effects are taken into account, including materials, micro-geometry and macro-geometry.

As far as BCT values are concerned, a correlation between the maximum compression load and the incipient buckling load was observed, the ratio between the two values being about two. However, due the few experimental values available, it is not possible to consider this relation general.

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